A LEO SATELLITE NAVIGATION ALGORITHM BASED ON GPS AND MAGNETOMETER DATA

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The Global Positioning System (GPS) has become a standard method for low cost onboard satellite orbit determination. The use of a GPS receiver as an attitude and rate sensor has also been developed in the recent past. Additionally, focus has been given to attitude and orbit estimation using the magnetometer, a low cost, reliable sensor. Combining measurements from both GPS and a magnetometer can provide a robust navigation system that takes advantage of the estimation qualities of both measurements. Ultimately, a low cost, accurate navigation system can result, potentially eliminating the need for more costly sensors, including gyroscopes. This work presents the development of a technique to eliminate numerical differentiation of the GPS phase measurements and also compares the use of one versus two GPS satellites.

INTRODUCTION

This work presents the testing of a unified navigation algorithm that produces estimates of attitude, angular rate, position, and velocity for a low earth orbit (LEO) spacecraft. The system relies on GPS phase and range data as well as magnetometer data. The algorithm used is an extended Kalman filter (EKF), blended with a 'pseudo-linear' Kalman filter algorithm, developed to provide LEO attitude, orbit,

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and rate estimates using magnetometer and sun sensor data[1]. For many LEO spacecraft the sun data is available during only a portion of the orbit. However, GPS data is available continuously throughout the orbit. GPS can produce accurate orbit estimates and combining GPS and magnetometer data improves the attitude and rate estimates[2]. The magnetometer based EKF can converge from large initial errors in position, velocity, and attitude[3]. Combining the magnetometer and GPS data into a single EKF provides a more robust and accurate system.

The algorithm is based on the EKF of Ref. 1. The GPS measurement models for phase and range are incorporated into the existing structure of the filter. The original EKF produced the orbit estimates in terms of Keplerian elements. Due to the nature of the GPS measurements and ease of computation, the orbit estimates are converted to the Cartesian position and velocity. The measurement model for the magnetometer is adjusted for this change in the state.

The estimation of the angular velocity, or rate, typically involves numerical differentiation that introduces noise into the update of the rate estimate. An alternative approach, called the 'estimation approach', avoids differentiation by incorporating the kinematics into the filter[4]. This method, applied to the *magnetometer*, is reported on in Ref. 5.

In the present work, the 'estimation approach' is also applied to the *GPS phase measurements*. Further testing with noisy sensor data is conducted on the EKF. Comparisons are made between the 'estimation approach' and the 'derivative approach' (requires numerical differentiation in the rate estimation[5]). Additionally, comparisons are made between estimates computed with a single GPS update and two GPS updates to determine if the additional complexity of two GPS updates provides a significant increase in the accuracy of the estimates.

ALGORITHM DEVELOPMENT

The GPS/magnetometer navigation system is based on an extended Kalman filter algorithm, blended with a 'pseudo-linear' Kalman filter originally presented in Ref. 1. (Further developments are presented in Ref. 5.) In the present work, the development of the 'estimation approach' as applied to the GPS phase measurements is presented. The EKF algorithm is summarized only. The general EKF equations are:

$$\underline{\dot{X}}(t) = \underline{f}(\underline{X}(t), t) + \underline{w}(t) \tag{1}$$

$$\underline{Z}_{k+1} = \underline{h}_{k+1}(\underline{X}(t_{k+1})) + \underline{n}_{k+1} \tag{2}$$

The general equations for the 'pseudo-linear' Kalman filter are:

$$\underline{\dot{X}}(t) = F(\underline{\hat{X}}(t_k))\underline{X}(t) + \underline{w}(t) \tag{3}$$

$$\underline{Z}_{k+1} = H_{k+1}\underline{X}_{k+1} + \underline{n}_{k+1} \tag{4}$$

The output filter state is given as

$$\underline{X}^T = [\underline{R}^T, \underline{V}^T, \underline{C}^T, q^T, \underline{\omega}^T, \underline{b}^T, \underline{z}^T] \tag{5}$$

Where $\underline{R}$ and $\underline{V}$ are the spacecraft position and velocity vectors, respectively, $\underline{C}$ is a vector of GPS receiver clock errors, q is the attitude quaternion, $\underline{\omega}$ is the rotation rate, $\underline{b}$ is the magnetic field in body coordinates (a result of using the 'estimation approach' with the magnetometer data[5]), $\underline{z}$ is the GPS phase measurement which contains either the phase measurement states for one GPS observation or for two. The state, $\underline{z}$, *is developed in this work.*

The filter processes data from the magnetometer, a 2-baseline GPS receiver, and reaction wheels (used in Euler's equations which describe the rotational dynamics of a spacecraft). As mentioned, the GPS data consists of range and phase measurements. Nominally, the filter first propagates the state estimate and filter covariance and then performs an update with the magnetometer data followed by an update with the GPS data. Data from either a single GPS satellite or from two GPS satellites are included in each update. The propagation of the state estimate and the filter covariance will be summarized first. The description of the state and covariance update will follow.

Filter Propagation

The dynamics for the position and velocity, the quaternion, and $\underline{\omega}$, $\underline{b}$, and $\underline{z}$ are treated as uncoupled. The position and velocity are propagated numerically with a two body gravity model which includes the j2 term. The clock errors are propagated according to a two state random-process model[6]. The quaternion is propagated numerically using the kinematics equation[7].

The rotation rate is propagated using Euler's equation. The states $\underline{b}$ and $\underline{z}$ are propagated with the equation expressing the time rate of change of a vector with respect to a rotating coordinate system. The equations for the rate and $\underline{b}$ are presented in Ref. 5. Following is the development of the equation expressing the time rate of change of the elements of $\underline{z}$.

The expression for a single GPS phase measurement is given as

$$z_{ij} = \vec{a}_i \cdot \vec{s}_j \tag{6}$$

where: z_{ij} = phase measurement
$\vec{a}_i$ = unit vector in direction of baseline i (i=1,2)
$\vec{s}_j$ = unit vector in direction of a GPS satellite (j=1 or 2)

Equation (6) can also be written as (dropping the subscripts)

$$z = \underline{a}^T \underline{s}_b = \underline{a}^T D \underline{s}_I \tag{7}$$

where: $\underline{a}$ = baseline direction expressed in body coordinates
D = direction cosine matrix, transforms from inertial to body coordinates

$\underline{s}_I$ = satellite to GPS direction expressed in inertial coordinates

Taking the derivative of (7) results in the following

$$\dot{z} = \underline{a}^T \dot{D} \underline{s}_I + \underline{a}^T D \dot{\underline{s}}_I \qquad (8)$$

The time derivative of the direction cosine matrix can be written as

$$\dot{D} = -[\underline{\omega} \times] D \qquad (9)$$

where: $[\underline{\omega} \times]$ = cross product matrix of the rotation rate

Substituting (9) into (8) and reordering the vectors gives

$$\dot{z} = \underline{a}^T [\underline{s}_b \times] \underline{\omega} + \underline{a}^T \hat{D} \dot{\underline{s}}_I \qquad (10)$$

where: $[\underline{s}_b \times] = [\hat{D} \underline{s}_I \times]$ = cross product matrix of the GPS direction unit vector in body coordinates

$\hat{D}$ = current estimate of the direction cosine matrix which transforms from inertial to body coordinates

$\dot{\underline{s}}_I$ = derivative of the GPS direction unit vector in inertial coordinates, this is not computed numerically but is developed from the relative position and velocity vectors (see Appendix)

The vector $\underline{z}$ is formed from the individual phase measurements, z_{ij} (either two components for the two baselines and one GPS satellite or four components for the two baselines and two GPS satellites). Equation (10) is then written in the general form as

$$\dot{\underline{z}} = F_z \underline{\omega} + \underline{u} \qquad (11)$$

Equation (11) is used to propagate $\underline{z}$ numerically.

In general, the linearized error dynamics are given as

$$\dot{\underline{x}}(t) = F \underline{x}(t) \qquad (12)$$

where: $F = \partial \underline{f} / \partial \underline{X}$

The full dynamics matrix, F, is given in References 1 and 5 for all the states except the phase measurement states, $\underline{z}$. The matrix F_z from (11) is augmented into the matrix F.

The filter covariance matrix is propagated according to

$$P_{k+1}(-) = \Phi_k(\hat{\underline{X}}_k(+)) P_k(+) \Phi_k{}^T(\hat{\underline{X}}_k(+)) + Q_k \qquad (13)$$

where: Φ = the state transition matrix = $e^{F\Delta t}$

Q = matrix of standard deviations of the white noise processes driving each of the states

The filter uses the 'multiplicative' approach in the attitude estimation[3], therefore the error state for the attitude is a vector of attitude errors.

Filter Update

The EKF state and covariance update equations are given as

$$\hat{\underline{X}}_{k+1}(+) = \hat{\underline{X}}_{k+1}(-) + K_{k+1}[\underline{y}_{k+1} - h_{k+1}(\hat{\underline{X}}_{k+1}(-)] \tag{14}$$

$$P_{k+1}(+) = [I - K_{k+1}H_{k+1}(\hat{\underline{X}}_{k+1}(-)]P_{k+1}(-)[I - K_{k+1}H_{k+1}(\hat{\underline{X}}_{k+1}(-)]^T + K_{k+1}RK_{k+1}^T \tag{15}$$

The Kalman gain is computed according to

$$K_{k+1} = P_{k+1}(-)H_{k+1}^T(\hat{\underline{X}}_k(-))[H_{k+1}(\hat{\underline{X}}_k(-))P_{k+1}(-)H_{k+1}^T(\hat{\underline{X}}_k(-)) + R_{k+1}]^{-1} \tag{16}$$

where $H = \partial\underline{h}/\partial\hat{\underline{X}}$

The measurement matrix, H, is partitioned as $H = [H_{orbit}, H_{attitude}, H_{rate,b,z}]$. The individual measurement matrices are defined according to the two measurements used, namely the magnetometer measurement and the GPS phase and range measurements. The development of the magnetometer and GPS measurement matrices are given in Ref. 5, only the development pertinent to the additional states, $\underline{z}$, is included here. The development of the GPS measurement matrix for the position and velocity states, presented in Ref. 5, includes both range and range-rate type measurements. In the current version of the EKF, only range measurements are included.

GPS Phase Measurement State Update

According to the development of the 'estimation approach' given in Ref. 4, the measured phase $\underline{z}_m$, is related trivially to the rate and the state $\underline{z}$ as

$$\underline{z}_m = \begin{bmatrix} 0 & 0 & I \end{bmatrix} \begin{bmatrix} \underline{\omega} \\ \underline{b} \\ \underline{z} \end{bmatrix} + \underline{\eta} = H_{rate,\underline{b},\underline{z}} \underline{X}_{rate,\underline{b},\underline{z}} + \underline{\eta} \tag{17}$$

where: $\underline{\eta}$ = added white noise

The innovations are then computed as

$$\underline{Z} = \underline{z}_m - H_{rate,\underline{b},\underline{z}} \underline{X}_{rate,\underline{b},\underline{z}} \tag{18}$$

and are used in the update equations for a linear Kalman filter. The $H_{rate,b,z}$ matrix replaces the portion of the GPS measurement matrix developed for the 'derivative approach' given in Ref. 5.

The use of the 'estimation approach' for GPS requires the initialization of the z states whenever a new GPS satellite is being observed. The states are initialized a priori with the first phase observations. The covariance is initialized with the a priori values and the correlation terms are set to zero.

SIMULATION AND TEST CONDITIONS

The test scenario consisted of an orbital ephemeris of the Rossi X-Ray Timing Explorer (RXTE) spacecraft and simulated orbits for the GPS constellation. The spacecraft is inertially pointing on an orbit whose average altitude is 575 km, its inclination is 57 degrees, and its eccentricity is 0.0015. The simulation generated from the RXTE ephemeris consists of 12 hours of sensor and reaction wheel data. After 6 hours, an attitude maneuver is simulated about the y axis at a rate of 0.2 deg/sec for 6.63 minutes (giving a total maneuver of 80 degrees). Two GPS antennas are aligned along the spacecraft x and y body axes and one at the origin, pointing in the $-z$ body axis direction. A magnetometer is aligned along the spacecraft body axes. The simulated magnetometer and GPS phase and range data contain added white noise, with standard deviations of 1 milliGauss, 1 degree, and 2 m, respectively. All the data are output at a rate of 2 Hz. The initial errors between the true state and the a priori filter state are given in Table I, except for the states $\underline{b}$ and $\underline{z}$. The states $\underline{b}$ and $\underline{z}$ are initialized with the first magnetometer and GPS phase measurements, respectively.

TABLE 1: INITIAL A PRIORI ERRORS IN THE FILTER STATE

STATE	ERROR
Position	1100 km (RSS)
Velocity	0.6 km (RSS)
Quaternion	111 deg rotation error
Rate	0.001 deg/sec/axis

RESULTS

Figures 1 and 2 show the RSS of the angular velocity errors for measurements from one and two GPS satellites, respectively, during the first 150 minutes of the data span. In both cases the estimation approach is used (as applied to the GPS phase measurements, recalling that the estimation approach is applied to the magnetometer). Here it can be seen that the use of two GPS satellites shortens the convergence time. After the transients have settled there is little difference between the two cases. The RSS rate errors using the derivative approach are slightly noisier, but show similar behavior to the estimation approach for both the one GPS and two GPS satellite scenarios. The jumps in the RSS errors within the first 50 minutes occur when there is a change to a different GPS satellite.
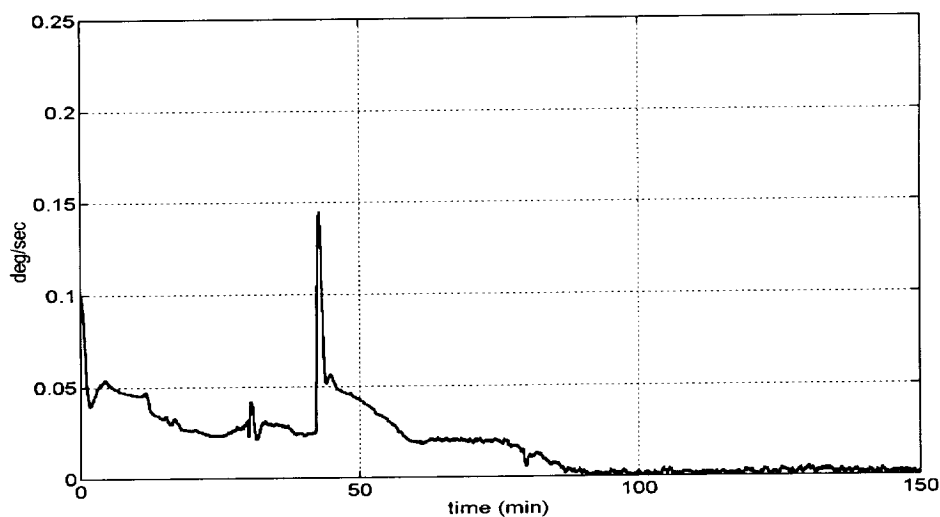
FIGURE 1. RSS Angular Velocity Error During First 150 Minutes, Measurements from One GPS Satellite
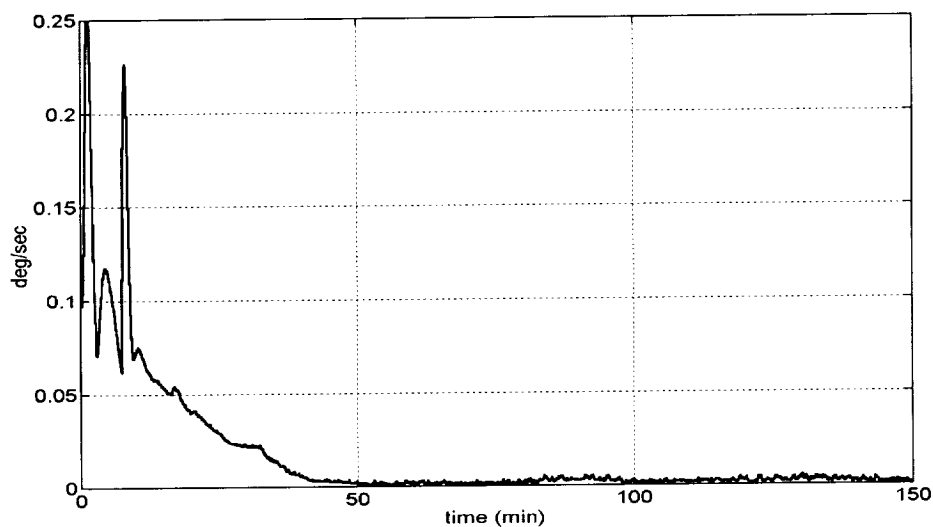

FIGURE 2. RSS Angular Velocity Error During First 150 Minutes, Measurements from Two GPS Satellites

Figures 3 and 4 show the estimated y angular velocity for the estimation and the derivative approaches, respectively, each using measurements from one GPS satellite. The time span in the figures is during the attitude maneuver, which occurs after 6 hours. While in Figures 1 and 2 we compared the effect of one and two GPS satellites (because the difference between the estimation and derivative approaches was negligible), in Figures 3 and 4 we compare the two approaches because here the difference stems from the approaches and the difference between using one or two GPS satellites is negligible. In both cases, the filter follows the maneuver. The other

two axes, namely the x and z axes, stay approximately at 0 deg/sec, similar to the estimated y rate both before and after the maneuver.
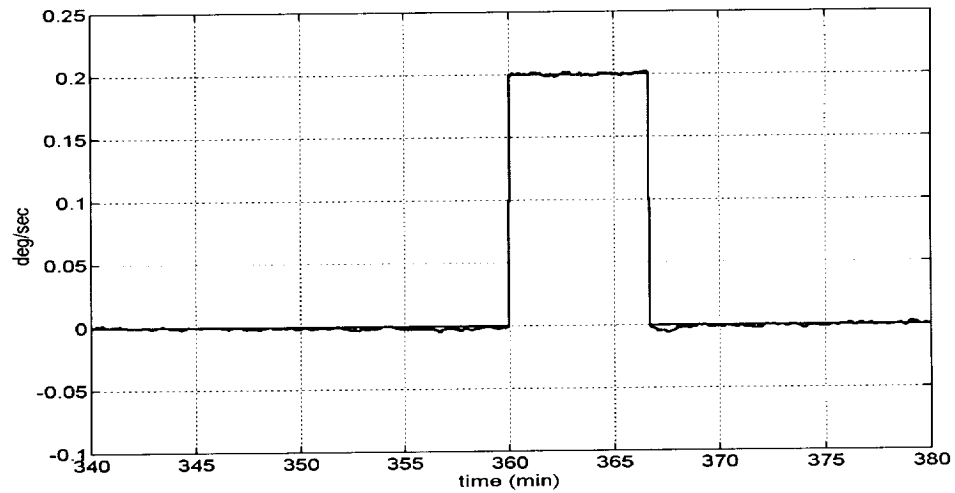

FIGURE 3. Estimated Angular Velocity About Y Axis (Estimation Approach)
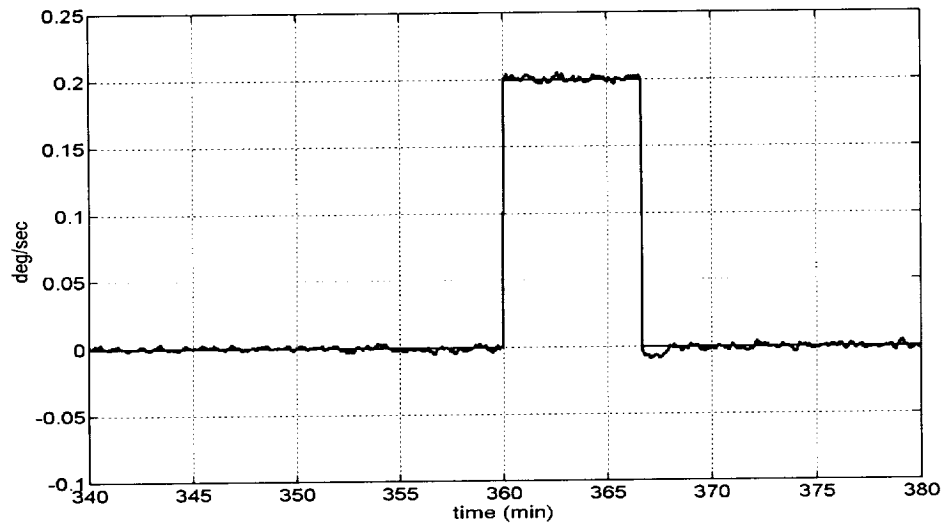

FIGURE 4. Estimated Angular Velocity About Y Axis (Derivative Approach)

Figures 5 and 6 show the RSS attitude error during the first 150 minutes, using measurements from one and two GPS satellites, respectively, using the estimation approach. Again, the use of two GPS satellites shortens the convergence time. After the filter has converged the results are similar with final average errors of approximately 0.3 degrees (computed over the last 30 minutes). The derivative approach results are similar to the estimation approach results.
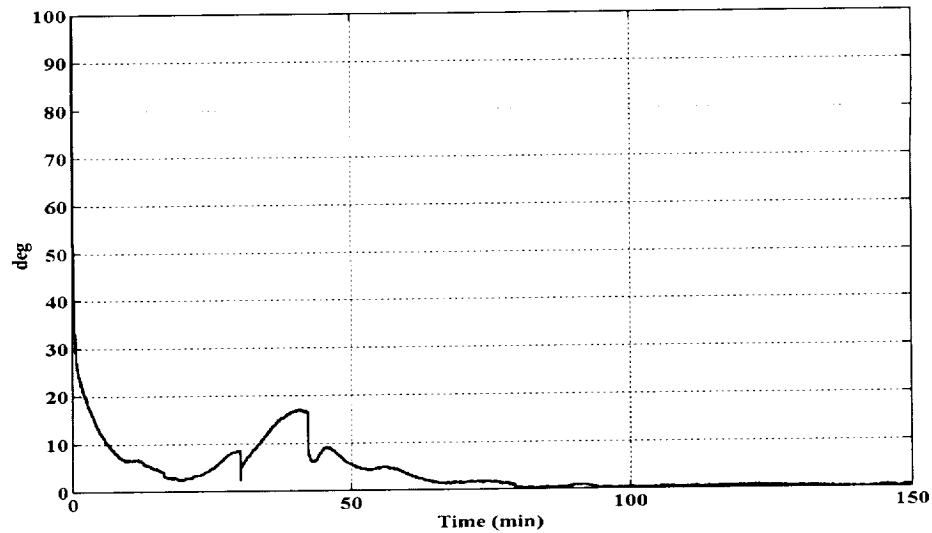
FIGURE 5. RSS Attitude Error During First 150 Minutes, Measurements from One GPS
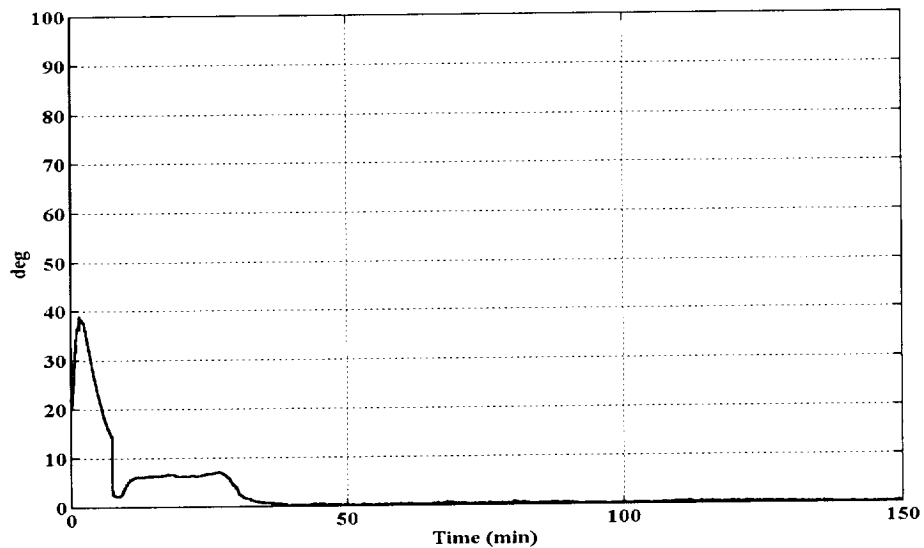
FIGURE 6. RSS Attitude Error During First 150 Minutes, Measurements from Two GPS

Finally, Figures 7 and 8 show the RSS position errors for measurements from one and two GPS satellites, respectively, using the estimation approach. Here, the difference between using one or two GPS satellites is the most significant (the same is true for the velocity errors which have plots similar in shape to the position errors). After 12 hours, the one GPS case is still converging, with peak errors of about 4.5 km and an average RSS over the last 30 minutes of 0.8 km. The spikes in the errors occur due to a change in the GPS satellite and there is also an error from the magnetic

field that occurs at the orbital period (approximately 90 minutes). In the case of using measurements from two GPS satellites, the effects of the changing GPS satellites are reduced. After 12 hours, the peak errors are reduced to less than 0.3 km with an average over the last 30 minutes of 30 m. The results for the final average RSS velocity errors are 6.6 cm/sec using two GPS versus 100 cm/sec when using one GPS.
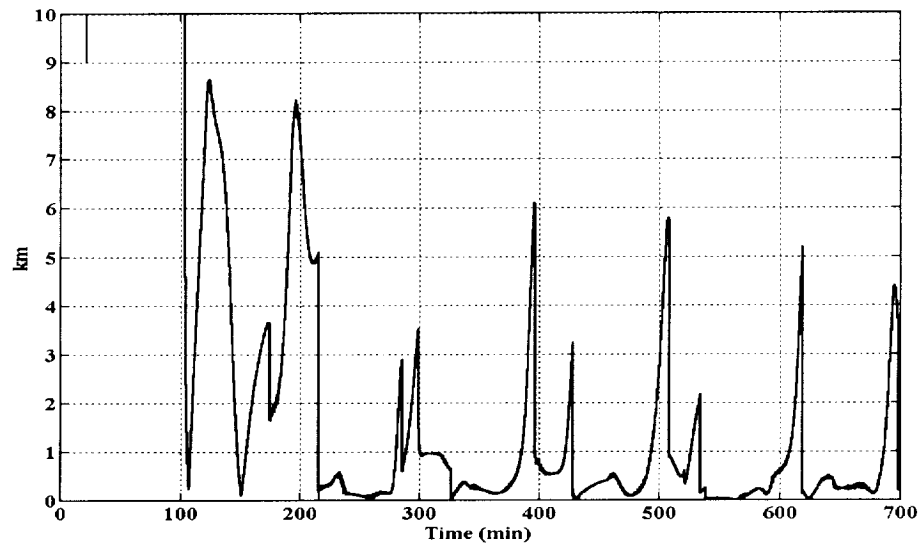

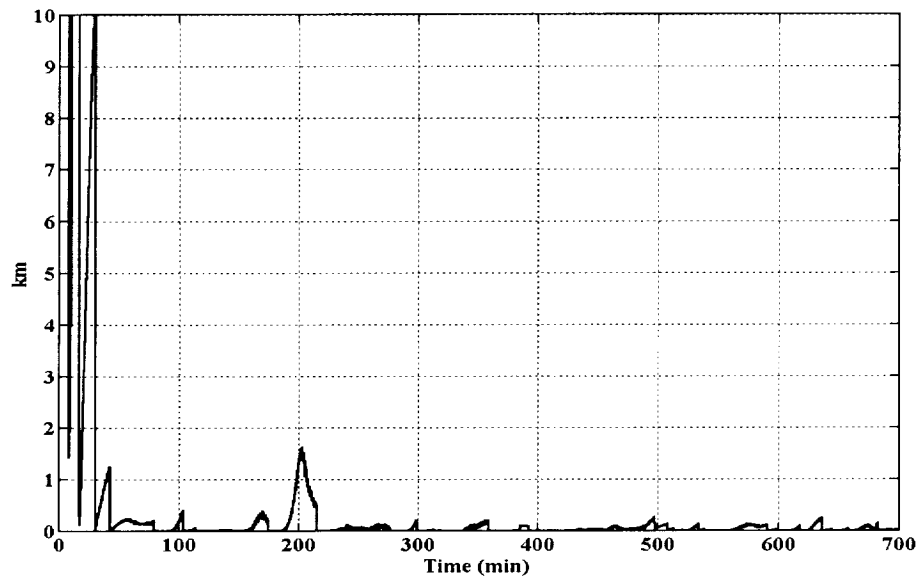FIGURE 7. RSS Position Error, Measurements from One GPS

FIGURE 8. RSS Position Error, Measurements from Two GPS

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3. Deutschmann, J., and I. Bar-Itzhack, "Comprehensive Evaluation of Attitude and Orbit Estimation Using Real Earth Magnetic Field Data", *Proceedings of the 11th Annual AIAA/USU Conference on Small Satellites*, Logan, UT, September 15-18, 1997.

4. Bar-Itzhack, I., "Classification of Algorithms for Angular Velocity Estimation", submitted to the *Journal of Guidance, Dynamics, and Control.*

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7. Wertz, James. R, editor, *Spacecraft Attitude Determination and Control*, D. Reidel Publishing Company, 1984.

APPENDIX - Development of the GPS Line of Sight Derivative

The position vector from the spacecraft to a GPS satellite is given as

$$R_s = R_{sv} - R \tag{A.1}$$

where R_{sv} = the position vector of the GPS satellite from the center of Earth
R = the position vector of the satellite from the center of Earth

The GPS line of sight unit vector is then given as

$$\vec{s} = \frac{R_s}{|R_s|} \tag{A.2}$$

Taking the derivative of $\vec{s}$ results in

$$\dot{\vec{s}} = \frac{d}{dt}\left(\frac{1}{|R_s|}\right)R_s + \frac{1}{|R_s|}\dot{R}_s \tag{A.3}$$

The first term on the right hand side of (A.3) can be written as

$$\frac{d}{dt}\left(\frac{1}{|R_s|}\right) = -\frac{R_s \cdot V_s}{|R_s|^3} \tag{A.4}$$

where: $V_s = V_{sv} - V = \dot{R}_s =$ the relative velocity of the GPS satellite with respect to the spacecraft

Substituting (A.4) into (A.3) results in

$$\dot{s} = -\frac{R_s \cdot V_s}{|R_s|^3} R_s + \frac{1}{|R_s|} V_s \tag{A.5}$$

The GPS position and velocity vectors R_{sv} and V_{sv}, respectively, are provided. However, only estimates of the spacecraft position and velocity vectors R and V, respectively, are available. Resolving all the vectors in inertial coordinates, and substituting in the estimated values results in the following expression for the derivative of the GPS line of sight vector implemented into the EKF algorithm.

$$\dot{\hat{s}}_I = -\frac{\hat{R}_{s,I}^T \hat{V}_{s,I}}{|\hat{R}_{s,I}|} \hat{R}_{s,I} + \frac{1}{|\hat{R}_{s,I}|} \hat{V}_{s,I} \tag{A.6}$$

where the ^ symbol refers to estimated values, and the 'I' refers to inertial coordinates.